Exhibit 21

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PASS-THROUGH AND THE PREDICTION OF MERGER PRICE EFFECTS*

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We use Monte Carlo experiments to study how pass-through can improve merger price predictions, focusing on the first order approximation (FOA) proposed in Jaffe and Weyl [2013]. FOA addresses the functional form misspecification that can exist in standard merger simulations. We find that the predictions of FOA are tightly distributed around the true price effects if pass-through is precise, but that measurement error in pass-through diminishes accuracy. As a comparison to FOA, we also study a methodology that uses pass-through to select among functional forms for use in simulation. This alternative also increases accuracy relative to standard merger simulation and proves more robust to measurement error.

I. INTRODUCTION

The profit maximizing level of cost pass-through in many standard oligopoly models depends on both the first and second derivatives of the consumer demand schedules. This insight dates back at least to Bulow and Pfleiderer [1983], and is extended and generalized in Weyl and Fabinger [2013] and Fabinger and Weyl [2015]. The more recent literature emphasizes that pass-through rates can be used to answer important questions in fields such as

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industrial organization, international trade, and mechanism design. An emerging empirical literature uses pass-through to study trade costs (e.g., Atkin and Donaldson [2015]), environmental regulation (e.g., Fabra and Reguant [2014]; Miller *et al.* [2015]), and health insurance (e.g., Cabral *et al.* [2014]).

In this article, we study how pass-through can inform predictions of merger price effects. One of the central purposes of antitrust analysis is to predict, with reasonable accuracy, the effect of mergers on prices. This has motivated the development of merger simulation techniques, which have been the subject of much academic work (e.g., Berry and Pakes [1993]; Hausman *et al.* [1994]; Werden and Froeb [1994]; Nevo [2000]) and have been implemented by practitioners at antitrust agencies and in the courtroom (Werden and Froeb [2007]).[1] The methodology relies on functional form assumptions about demand, under which post-merger equilibrium is computed. It is well established that predictions are sensitive to these assumptions (e.g., Werden [1996]; Crooke *et al.* [1999]). Because the functional forms implicitly fix the second order properties of demand, and because pass-through is driven in part by these second order properties, there is a theoretical basis for thinking that observed pass-through could ameliorate prediction error caused by functional form misspecification.

We focus on the theoretical finding of Jaffe and Weyl [2013] that a first order approximation (FOA) to post-merger prices can be calculated given knowledge of the first and second derivatives of demand. Provided that elasticities can be estimated or calibrated, FOA can be implemented by inferring the second derivatives of demand from pass-through. The first order effects of the merger then are calculated with little reliance on functional form assumptions.[2] Jaffe and Weyl [2013] prove that FOA is precise for arbitrarily small price changes – here we extend the analysis to mergers with wide-ranging price effects. As a point of comparison for FOA, we also explore a method that we refer to as 'informed simulation,' in which a demand system is selected that elicits pass-through close to what is observed (Miller *et al.* [2013]). Simulation then is conducted with the selected demand system. As we demonstrate in this article, both FOA and informed simulation are more accurate than 'standard' merger simulation in which demand schedules are selected without regard for pass-through.

Our findings rely on Monte Carlo experiments. We generate a data set comprised of a large number of markets in which the underlying demand system is either logit, linear, almost ideal, or log-linear. These four demand systems allow for a wide range of curvature and pass-through conditions, and are commonly employed in antitrust analyses of mergers involving

[1] In merger investigations, simulation often is used to complement other evidence, including documentary evidence and reduced-form empirical work of the type presented in the Staples/Office Depot trial (Dalkir and Warren-Boulton [2004]).

[2] In settings that involve more than two firms, a modified horizontality condition is useful in interpreting pass-through information. We discuss the details in Section II(ii).

differentiated products (Werden *et al*. [2004]; Werden and Froeb [2007]). They have also been used in academic studies that examine the effect of demand curvature on the precision of counterfactual simulations (e.g., Crooke *et al*. [1999]; Huang *et al*. [2008]). We alternately consider scenarios in which pass-through is observed perfectly, with measurement error, and with systematic bias.

We find that FOA dominates standard merger simulation, provided that pass-through is observed perfectly and there is some functional form misspecification in the simulation.[3] The predictions of FOA are tightly distributed around the true price effects. The median absolute prediction error (MAPE) that arises with FOA typically is a fraction of the MAPE with standard merger simulation, and FOA is more accurate in 93% of the merger scenarios considered. Further, when price effects are evaluated against a specific ten per cent threshold, FOA produces far fewer false positives and false negatives than standard merger simulation. These results demonstrate that having accurate information on pass-through can greatly improve the accuracy of counterfactual predictions.

We also find, however, that the accuracy of FOA deteriorates as measurement error in pass-through is incorporated into the experiments. When pass-through is observed within 90% of its true value, the MAPEs that arise with FOA and standard merger simulation are of similar magnitudes, and if functional form specification also is minor then simulation tends to be more accurate than FOA.[4] The relative accuracy of FOA is preserved with more modest measurement error. Finally, we find that upward bias in pass-through causes FOA to over-predict price increases, and downward bias leads to under-predictions. This sensitivity arises because FOA uses pass-through to infer demand curvature, so if pass-through is observed with error then this feeds directly into the price predictions. Taken together, our results show that FOA requires *precise* information on pass-through behavior in order to give accurate results.

This does not imply that noisy pass-through should be discarded. To the contrary, we find that informed simulation also outperforms standard merger simulation, and that it is relatively robust to measurement error in pass-through. While FOA typically is more accurate than informed simulation when pass-through is observed perfectly, the MAPEs that arise with informed simulation and FOA are roughly equal when pass-through is observed within 60% of its true value, and informed simulation is more

[3] We assume throughout that demand elasticities in the pre-merger equilibrium are known with certainty. Thus, absent misspecification, simulation generates the post-merger prices exactly. The comparison of FOA to misspecified merger simulation is informative because the underlying demand schedules in most real-world markets are unlikely to conform to any of the standard models, so that functional form misspecification is prevalent in merger simulation.

[4] An example of a minor functional form misspecification would be a simulation with linear demand when the true demand system is logit.

accurate when pass-through is observed within 90% of its true value. In each of these scenarios, informed simulation is more accurate than standard merger simulation. Robustness to measurement error derives from the way in which pass-through affects the predictions of informed simulation: pass-through has no direct effect because it is used only to select among demand schedules. This limits the influence of poorly measured pass-through terms that are difficult to reconcile with economic theory. The finding suggests that it is appropriate to interpret pass-through using an economic model if it is observed with significant measurement error.

A number of caveats apply. First, the experimental design limits the accuracy of informed simulation. Given perfect knowledge of pass-through, it is possible to identify the correct demand system with which to perform simulation, and thereby recover post-merger prices exactly. We view this as unrealistically optimistic because, in practice, consumer decisions need not align with *any* of the models used in our experiments. Thus, to implement informed simulation, we identify the misspecified demand system that produces pass-through closest to what is observed, and simulate using that demand system. The approach makes informed simulation less accurate than FOA in the presence of perfect pass-through information. The extent to which this extends to practical settings depends on how closely observed pass-through mimics what can be generated by an economic model. The findings that (i) informed simulation is more accurate than standard merger simulation and (ii) informed simulation handles measurement error better than FOA should be more robust.

Additionally, we note that the data generating process used in the Monte Carlo experiments cannot be expected to reflect perfectly the conditions of real-world markets. The magnitude of prediction error that arises due to functional form misspecification, in particular, is driven by our reliance on the logit, linear, almost ideal, and log-linear demand systems. We nonetheless consider the results to be valuable, as they extend the theoretical insights of Jaffe and Weyl [2013] beyond arbitrarily small mergers, and they inform the way in which pass-through can best be used to improve counterfactual predictions. Some of the accuracy gains we document can be achieved by using the random coefficients logit (RCL) demand system, which is theoretically flexible enough to match the elasticities and curvature of the true underlying demand system (e.g., as in Nevo [2000]). Because supply-side variation often identifies the nonlinear demand parameters, the RCL can be interpreted as another methodology that allows pass-through to inform predictions.[5]

[5] In many applications, the flexibility afforded by the RCL is limited due to a sparse representation of consumer indirect utility (e.g., Hellerstein [2008]; Nakamura and Zerom [2010]; Miller and Weinberg [2015]). For example, a specification that incorporates only unobserved heterogeneity in the price coefficient does not allow the elements of the pass-through matrix to shift independently of one another. Standard merger simulations employing simpler demand systems, rather than the RCL, tend to be used in antitrust enforcement due to time constraints and the computational demands of RCL estimation.

The paper proceeds as follows. First, we outline the theoretical framework in Section II. The focus is on mergers in differentiated-products Nash-Bertrand models, and we develop the means by which pass-through can be used to inform prediction following Jaffe and Weyl [2013]. Section III provides the details of the Monte Carlo experiments. Section IV presents summary statistics on pass-through and the merger price effects that arise in the data. Section V develops the results regarding whether and how pass-through can improve counterfactual predictions. In Section VI, we summarize and sketch some thoughts regarding the difficulties that can arise in obtaining and interpreting estimates of pass-through.

II. THEORETICAL FRAMEWORK

II(i). *Merger Price Effects*

We examine mergers in the context of a Bertrand-Nash oligopoly model of price competition among multi-product firms. Mergers change the unilateral pricing calculus of the merging firms and, provided that products are substitutes and countervailing merger efficiencies are small, result in a new equilibrium characterized by higher prices. Assume that each firm faces a well-behaved, twice-differentiable demand function. The equilibrium prices of each firm $i \in I$ satisfy the following first order conditions:

$$(1) \qquad f_i(P) \equiv -\left[\frac{\partial Q_i(P)^T}{\partial P_i}\right]^{-1} Q_i(P) - P_i + MC_i(Q_i(P)) = 0 \quad \forall i \in I$$

where P_i is a vector of firm i's prices, $Q_i(P)$ is a vector of firm i's unit sales, P is a vector containing the prices of every product, and MC_i is the marginal cost function. Consider a merger between firms j and k that, for simplicity, does not affect the marginal cost and demand functions. The first order condition changes such that:

$$(2) \qquad h_i(P) \equiv f_i(P) + g_i(P) = 0 \quad \forall i \in I$$

where

$$(3) \qquad g_j(P) = -\underbrace{\left(\frac{\partial Q_j(P)^T}{\partial P_j}\right)^{-1}\left(\frac{\partial Q_k(P)^T}{\partial P_j}\right)}_{\text{Matrix of Diversion from } j \text{ to } k}\underbrace{(P_k - MC_k^1)}_{\text{Markup of } k}$$

and $g_k(P)$ is defined analogously, while $g_i(P) = 0$ for all $i \neq j, k$. The g function is the product of firm k's markups and the matrix of diversion ratios between firms j and k, which depend upon the first derivatives of the

demand functions. Equation (3) captures an opportunity cost created by the merger: each merging firm, when making a sale, possibly forgoes a sale of its merging partner (Farrell and Shapiro [2010]).[6]

The prices that satisfy equation (2) depend on how the demand and marginal cost functions change as prices move away from the pre-merger equilibrium. Nonetheless, the first order effects of the merger depend only on information that is local to the pre-merger equilibrium (Jaffe and Weyl [2013]). Specifically, a first order approximation (FOA) to the price changes that arise from the merger is given by:

$$\text{(4)} \qquad \Delta P = -\left(\frac{\partial h(P)}{\partial P}\right)^{-1}\Bigg|_{P=P^0} g(P^0)$$

where P^0 is the vector of pre-merger prices. The first order effects therefore depend upon the opposite inverse Jacobian of $h(P)$, which Jaffe and Weyl [2013] refer to as *merger pass-through* matrix. This matrix incorporates both the first and second derivatives of demand, and can be conceptualized as the rate at which the change in pricing incentives from the merger are transmitted to consumers. Therefore, when using equation (2) or (4) to infer the price changes that arise from a merger, the accuracy of the inference depends on how well the higher-order properties of real-world demand are captured.

II(ii). *Pass-Through and Prediction*

Merger simulation is one methodology in the industrial organization literature used to predict the price effects from a merger (Nevo and Whinston [2010]). It requires functional forms for the demand and marginal cost functions to be selected and parameterized, which in turn allows post-merger prices to be computed as the solution to the post-merger first order conditions.[7] Because the assumed functional forms implicitly restrict the second derivatives of demand, misspecification bias can arise even if the demand function captures perfectly the elasticities (i.e., the first derivatives) that arise in the pre-merger equilibrium.[8]

[6] The g function is referred to in the antitrust literature as upward pricing pressure (UPP).

[7] In practice, elasticities are typically obtained through demand estimation or calibration. A substantial literature focuses on the conditions under which regression analysis recovers consistent estimates of consumer substitution (e.g., Berry *et al.* [1995]; Nevo [2000]). Elasticities alternatively could be calibrated to match price-cost margins and customer switching patterns, as is more common in merger enforcement (e.g., Remer and Warren-Boulton [2015]).

[8] For many common demand systems, the second derivatives are fully determined by the elasticities. This is the case for the linear, logit, nested logit, almost ideal, and log-linear demand systems. The random coefficient logit model is theoretically capable of divorcing the first and second derivatives, but in most applications the specification employed results in only a limited amount of flexibility.

We explore the extent to which pass-through can be used to inform the second derivatives of demand. Jaffe and Weyl [2013] demonstrate, through an application of the implicit function theorem, that the cost pass-through matrix in pre-merger equilibrium is given by:

$$\rho(P)|_{P=P_0} = -\left(\frac{\partial f(P)}{\partial P}\right)^{-1}\Bigg|_{P=P_0} \tag{5}$$

Thus, pass-through equals the opposite inverse of the pre-merger first order conditions, and it depends directly on both the first and second derivatives of demand. It follows that, given demand elasticities, equation (5) provides a mapping between pass-through and the second derivatives. This allows for the second derivatives of demand to be imputed from pass-through, and used to calculate FOA as proposed in Jaffe and Weyl [2013]. Alternatively, equation (5) can be used to obtain the pass-through rates that arise under different candidate demand systems. Then, an informed simulation can be conducted using the functional form of demand that generates pass-through close to the observed pass-through rates (Miller *et al.* [2013]).[9] Either approach operates to mitigate misspecification error.

Because the number of second derivatives exceeds the number of pass-through terms, restrictions in addition to equation (5) are needed to identify the full set of second derivatives. This is relevant if one is attempting to calculate FOA based on cost pass-through. Slutsky symmetry is sufficient to identify all second derivatives for duopoly markets. If there are more than two firms, then second derivatives of the form $\partial^2 Q_i/(\partial P_j \partial P_k)$, for $i \neq j$, $i \neq k$ and $j \neq k$, remain unidentified without further restrictions. As suggested in Jaffe and Weyl [2013], the following assumption is sufficient:

$$\frac{\partial^2 Q_i}{\partial P_j \partial P_k} = \frac{\partial^2 Q_i}{\partial^2 P_i}\frac{\frac{\partial Q_i}{\partial P_j}\frac{\partial Q_i}{\partial P_k}}{\left(\frac{\partial Q_i}{\partial P_i}\right)^2} \qquad (i \neq j, i \neq k, j \neq k) \tag{6}$$

This restriction is exact only if demand adheres to a modified horizontality condition.[10] Thus, imputation based on equation (6) itself can introduce misspecification error. However, because error is only introduced for a limited subset of second derivative terms, one might expect this to be

[9] We define the distance metric that we use to evaluate 'closeness' in the next section. Informed simulation still requires that a menu of candidate demand systems be selected, and the results can depend on which demand systems are included.

[10] The condition, proposed in Jaffe and Weyl [2013], is that $Q_i(P) = \psi\left(P_i + \sum_{j \neq i} \mu_j(P_j)\right)$ for some $\psi : \mathbb{R} \rightarrow \mathbb{R}$ and $\mu : \mathbb{R} \rightarrow \mathbb{R}$. Among the four demand systems considered later in this paper, only linear demand satisfy the condition precisely.

inconsequential relative to the misspecification error that may arise with simulation. Indeed, the Monte Carlo evidence we develop indicates that the loss of predictive accuracy that arises with this imputation tends to be small.[11]

III. MONTE CARLO EXPERIMENTS

III(i). *Overview*

In the remainder of the paper, we present numerical evidence regarding the extent to which cost pass-through information can improve the accuracy of counterfactual predictions, including instances in which pass-through is observed with measurement error or bias. All of the numerical experiments take as given the demand elasticities that arise in the pre-merger equilibrium. We focus instead on perturbing what is known about demand curvature, as revealed through pass-through.[12]

We work with the logit, almost ideal, linear, and log-linear demand systems. Because the curvature properties of these systems are fully determined by the elasticities, we can calibrate them such that the first derivatives are identical across the demand systems in the pre-merger equilibrium but the curvature (and pass-through) conditions differ. This conveys tractability to the data generating process and facilitates comparisons across demand systems. Given the theoretical relationship between demand curvature and the magnitude of merger price effects, a reasonable hypothesis is that FOA and informed simulation should outperform standard merger simulations if the observed pass-through information is of sufficiently high quality. Our experiments largely confirm this hypothesis. Importantly, we are able to quantify both how much pass-through can improve predictive accuracy and how quickly improvements diminish as measurement error and bias are introduced to the observed pass-through rates.

III(ii). *Data Generating Process*

We generate simulated data that comport with the theoretical assumptions outlined previously. The markets feature four firms that produce differentiated products with a constant returns-to-scale production technology. Competition is in prices and equilibrium is Bertrand-Nash. Each draw of data is independent and characterizes the conditions of a single market,

[11] In Appendix Figure C.1 (See Supplementary Materials online), we use scatter-plots to compare the predictions of FOA calculated based on equation (6) to FOA predictions calculated with perfect knowledge of the second derivatives. FOA predictions across the two approaches are nearly identical with logit, almost ideal, and linear demand, and remain similar with log-linear demand. It follows that imputation under the modified horizontality condition does not create meaningful misspecification error in our experiments.

[12] We use the term 'demand curvature' interchangeably with second derivatives of demand.

and the simulated data cover a wide range of competitive conditions that derive from the randomized draws. We normalize all prices to unity in the pre-merger equilibrium, which conveys the advantage that merger effects are the same in levels and percentages.[13] The details of the data generating process are as follows:

1. Randomly draw (i) market shares for four firms and an outside good, and (ii) the first firm's margin based on a uniform distribution bounded between 0.20 and 0.80.
2. Calibrate the parameters of a logit demand system based on the margin and market shares, and calculate the demand elasticities that arise in the pre-merger equilibrium. This entails selecting demand parameters that rationalize the random data. The parameters are exactly identified given market shares, prices, and a single margin.
3. Calibrate linear, almost ideal, and log-linear demand systems based on the logit demand elasticities. The parameters of these systems are exactly identified given market shares, prices, and the logit demand elasticities.[14]
4. Simulate the price effects of a merger between two firms under each of the demand systems.
5. Repeat steps (1) - (4) until 3,000 draws of data are obtained.

The algorithm generates 12,000 mergers to be examined, each defined by a draw of data and a demand system.[15] As discussed above, the data generating process requires that pre-merger demand elasticities are identical across demand systems for a given draw of data. We provide mathematical details on the calibration process in Appendix A.

The data generating process allows us to isolate the role of demand curvature in driving merger price effects and to explore cleanly how curvature assumptions matter for simulation. For instance, consider a merger defined by a given draw of data and the logit demand system. The true price effect of the merger is obtained from a logit simulation, and this can be compared against simulation results obtained under alternative assumptions of almost ideal, linear, and log-linear demand. The existing literature indicates

[13] The loss of generality caused by the price normalization is limited, and we have confirmed that alternatives do not affect results.

[14] In the pre-merger equilibrium, consumer substitution between products is proportional to market share because all the systems are calibrated based on logit elasticities. This reduces the dimensionality of the random data that must be drawn. The substitution-by-share property is retained away from the pre-merger equilibrium only for logit demand.

[15] A small number of draws cannot be rationalized with logit demand – this arises if the first firm has both an unusually small market share and an unusually high price-cost margin. We replace these to obtain the 3,000 draws. The data generating process also produces some markets that exhibit extreme pass-through conditions, and others with no post-merger equilibria. We exclude those calibrations from the analysis, treating as extreme a pass-through rate that is negative or exceeds ten. The pass-through criterion eliminates 74 almost ideal markets and 164 log-linear markets. We do not redraw these markets.

that prediction error due to functional form misspecification along these lines is substantial (e.g., Crooke *et al.* [1999]), and this sensitivity is consistent with the theoretical results provided in Section II.

To assess the extent to which pass-through improves predictive accuracy, we posit first that the cost pass-through matrix is available for use without measurement error or bias.[16] We then calculate FOA based on equation (4), using the horizontality restriction of equations (5) and (6) to impute second derivatives.[17]

We also evaluate an 'informed simulation' in which pass-through is used to select an appropriate demand system. Given perfect knowledge of pass-through, as posited initially, and the design of our experiments, it is possible to identify the correct demand system with which to perform prediction. We view this as unrealistically optimistic because, in practice, consumer decisions need not align with *any* of the models used in our experiments. It follows that testing the accuracy of informed simulation should involve the mitigation, but not the complete elimination, of functional form misspecification. Thus, in our implementation, we identify the misspecified demand system that produces pass-through that is closest to what is observed, and simulate using that demand system.[18]

These results in hand, we next incorporate measurement error and bias into the observed pass-through data, and evaluate how the predictive accuracy of FOA and informed simulation changes. To add noise, we add a uniformly distributed error to each element of the pass-through matrix. Mathematically, we define the observed pass-through element (j,k) to be

$$(7) \qquad \tilde{\rho}_{jk} = \rho_{jk} + \epsilon \quad \text{where} \quad \epsilon \sim U(\rho_{jk} - t\rho_{jk}, \rho_{jk} + t\rho_{jk})$$

The support of the error is element-specific and depends on t. We use three different levels for t, such that pass-through is observed alternately within 30, 60, and 90 per cent of its true value. To add bias, we suppose that what

[16] We obtain the pre-merger cost pass-through matrix from equation (5), using second derivatives that are obtained analytically from the calibrated demand systems (see Miller *et al.* [2013]).

[17] To be clear, we use the actual second derivatives to obtain the pass-through matrices. Then we assume that pass-through, but not the second derivatives, is available for use in prediction. With FOA, the imputed second derivatives match the actual second derivatives exactly only for the case of linear demand, due to the misspecification bias that otherwise is introduced by the modified horizontality restriction.

[18] We use mean squared error as the distance measure. Let ρ_{jk} be the (j,k) element of the observed pass-through matrix, and let $\hat{\rho}^i_{jk}$ be the analog for demand system i. The mean squared error for demand system i is given by $MSE_i = \sum_{j,k} (\rho_{jk} - \hat{\rho}^i_{jk})^2$. It is not necessary to observe the full pass-through matrix to support informed simulation. Indeed, our experiments indicate that prediction error is comparable if instead industry pass-through is used to select among demand systems.

TABLE I
ORDER STATISTICS

	Median	5%	10%	25%	75%	90%	95%
Characteristics Invariant to Demand Form							
Market share	0.21	0.03	0.06	0.13	0.28	0.35	0.40
Margin	0.48	0.23	0.26	0.34	0.62	0.72	0.76
Elasticity	2.08	1.32	1.38	1.60	2.94	3.91	4.38
Own-Cost Pass-Through							
Logit	0.80	0.63	0.67	0.73	0.88	0.94	0.97
AIDS	1.19	0.75	0.78	0.90	1.72	2.36	2.82
Linear	0.53	0.51	0.51	0.52	0.55	0.57	0.58
Log-Linear	1.87	1.29	1.34	1.50	2.52	3.39	3.98
Cross-Cost Pass-Through							
Logit	0.04	0.00	0.01	0.02	0.06	0.09	0.11
AIDS	0.22	0.03	0.06	0.12	0.39	0.70	0.98
Linear	0.09	0.01	0.02	0.05	0.12	0.15	0.17
Log-Linear	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Industry Pass-Through							
Logit	0.95	0.84	0.87	0.91	0.97	0.99	0.99
AIDS	1.90	1.10	1.18	1.39	2.92	4.32	5.27
Linear	0.79	0.67	0.70	0.74	0.87	0.94	0.97
Log-linear	1.87	1.29	1.34	1.50	2.52	3.39	3.98
Merger Price Effects							
Logit	0.09	0.01	0.02	0.05	0.16	0.24	0.30
AIDS	0.18	0.01	0.03	0.08	0.46	1.09	1.88
Linear	0.08	0.01	0.02	0.04	0.14	0.21	0.28
Log-Linear	0.30	0.02	0.05	0.12	0.77	2.08	4.11

Notes: Summary statistics are based on 3,000 randomly-drawn sets of data on the pre-merger equilibria. The market share, margin, and elasticity are for the first firm. Market share and margin are drawn randomly in the data generating process, while the elasticity is the own-price elasticity of demand and equals the inverse margin. Pass-through is calculated, following calibration, based on the curvature properties of the respective demand systems. Own-cost pass-through is the derivative of firm 1's equilibrium price with respect to its own marginal cost. The cross-cost pass-through statistics are based on the derivative of firm 1's equilibrium price with respect to firm 2's marginal cost. The merger price effects are the change in firm 1's equilibrium price.

is observed for each element (j, k) is $\tilde{\rho}_{jk} = \rho_{jk}(1+s)$, where we set $s = \pm 0.15$ to reflect some degree of upward or downward bias.[19]

IV. SUMMARY STATISTICS

In Table I, we summarize the empirical distributions that arise in the data. The market shares and margins of firm 1 are obtained from random draws. Because shares are allocated among the four products and the outside good, the distribution of firm 1's share is centered around 20 per cent. The margin distribution reflects uniform draws with support over (0.20, 0.80). The own-price elasticity of demand, which equals the inverse margin, has a

[19] Recent research demonstrates that standard orthogonality conditions are insufficient to ensure that reduced-form regressions of prices on cost shifters yield unbiased estimates of pass-through (MacKay *et al*. [2014]). Bias arises, for example, if pass-through is not constant in prices and the cost distribution is asymmetric.

distribution centered around 2.08, and 90 per cent of the elasticities fall between 1.32 and 4.38. These statistics are invariant to the posited demand system because the demand systems are calibrated to reproduce the same first-order characteristics in the pre-merger equilibria.

Cost pass-through depends on demand curvature and varies across the four demand systems. We define *own pass-through* as the effect of an individual firm's costs on its equilibrium price. The own pass-through terms fall along the diagonal of the pass-through matrix. Median own pass-through equals 0.80, 1.19, 0.53, and 1.87 for the logit, almost ideal, linear, and log-linear demand systems, respectively. Own-cost pass-through has wide support for the almost ideal and log-linear demand systems but is more tightly distributed for the logit and (especially) the linear demand systems. We define *cross pass-through* as the effect of a specific competitor's cost on an equilibrium price – cross pass-through is isomorphic to strategic complementarity in prices (Bulow *et al*. [1985]). The cross pass-through terms are the off-diagonal elements of the pass-through matrix. Median cross pass-through equals 0.04, 0.22, 0.09, and 0.00 across the four demand systems. Thus, while the almost ideal and log-linear demand systems both tend to generate large own pass-through, only the AIDS generates large cross pass-through because prices are not strategic complements (or substitutes) with log-linear demand.

We also report statistics for *industry pass-through*, which we define as the effect on equilibrium prices of a cost increase that is experienced by all firms. While knowledge of industry pass-through alone is insufficient to obtain an FOA to a merger price effect, it can inform counterfactual prediction in some simpler settings. Further, much of the existing empirical literature relies on industry-wide cost changes for identification, such as exchange rate fluctuations (e.g., Gopinath *et al*. [2011]), sales taxes (e.g., Barzel [1976]), and input prices (e.g., Genesove and Mullin [1998]). Our data inform the levels of industry pass-through that one might expect to estimate with reduced-form techniques, absent trade costs and other market frictions. As shown, median industry pass-through equals 0.95, 1.90, 0.79, and 1.87 for the logit, almost ideal, linear, and log-linear demand systems, respectively. Industry pass-through will always exceed own pass-through if prices are strategic complements, as is the case for three of our demand systems.

The median merger price effects are 0.09, 0.18, 0.08, and 0.30 for the logit, almost ideal, linear and log-linear demand systems, respectively. Because pre-merger prices are normalized to one, these statistics reflect both the median level change and median percentage change. Dispersion within demand systems mainly reflects the range of upward pricing pressure that arises from the data generating process. Dispersion across demand systems reflects the specific pass-through properties of the systems, with greater own pass-through associated with larger price effects. This

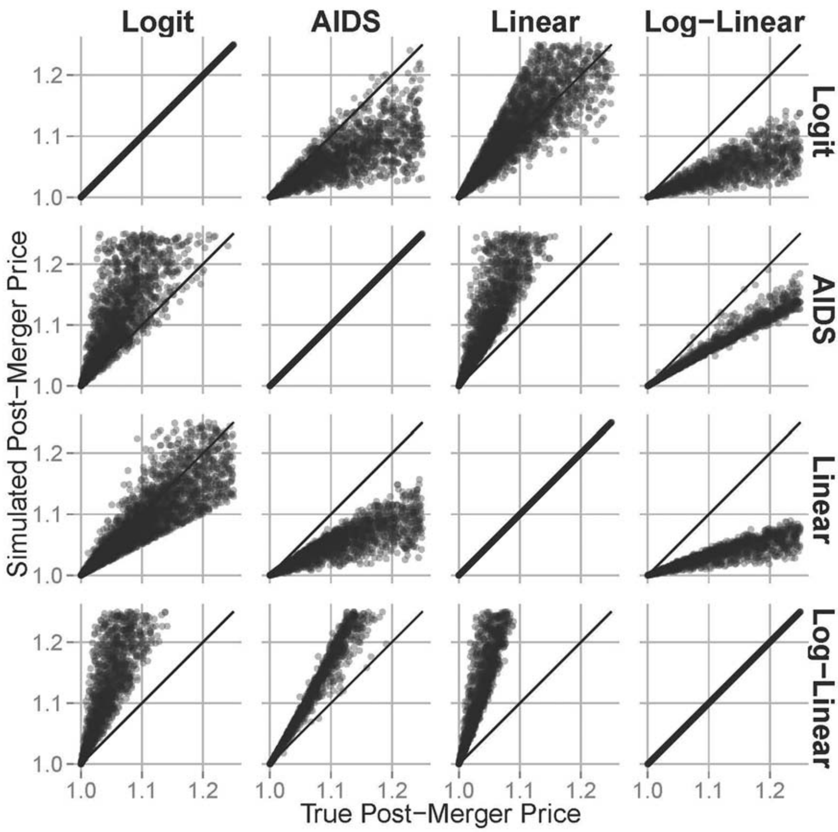

Figure 1
Prediction Error from Standard Merger Simulations

Notes: The scatter plots characterize the accuracy of merger simulations when the underlying demand system is logit (column 1), almost ideal (column 2), linear (column 3) and log-linear (column 4). Merger simulations are conducted assuming demand is logit (row 1), almost ideal (row 2), linear (row 3), and log-linear (row 4). Each dot represents the first firm's predicted and actual post-merger prices for a given draw of data.

relationship, first observed in Froeb *et al.* [2005], is explained by the theoretical results of Jaffe and Weyl [2013].

Figure 1 further explores how functional form assumptions affect the predictions of simulation. The scatter plots characterize the accuracy of merger simulations when the underlying demand system is logit (column 1), almost ideal (column 2), linear (column 3), and log-linear (column 4). Merger simulations are conducted assuming demand is logit (row 1), almost ideal (row 2), linear (row 3), and log-linear (row 4). Each dot represents the predicted and true changes in firm 1's price for a given draw of data; its vertical position is the prediction of simulation and its horizontal

position is the true price effect. Dots that fall along the 45-degree line represent exact predictions while dots that fall above (below) the line represent over (under) predictions. Prediction error is zero when the functional form used in simulation matches that of the underlying demand system.[20]

As shown, logit and linear simulations under-predict the price effects of mergers when the underlying demand system is almost ideal or log-linear. AIDS simulation over-predicts price increases when the underlying demand system is logit or linear but under-predicts when it is log-linear. Log-linear simulation over-predicts price increases in all cases. This sensitivity of prediction to functional form assumptions is well known (e.g., Crooke *et al.* [1999]) and, in antitrust settings, it is standard practice to generate predictions under multiple different assumptions as a way to evaluate the scope for price changes. We explore next the extent to which cost pass-through can be used to improve the precision of merger predictions.

V. RESULTS

V(i). *Perfect Information on Pass-Through*

Figure 2 provides scatter plots of the prediction error that arises with FOA and informed simulation, for the cases in which pass-through is observed precisely. As shown, FOA yields accurate predictions when the underlying demand system is logit or almost ideal, as demonstrated by the clustering of dots around the 45° line. It is exact with linear demand, as it is in any setting that produces a quadratic profit function. Prediction error is somewhat larger with the log-linear demand system. Informed simulation provides noisier estimates than FOA, but the biases are reduced when compared with standard merger simulations. These results are consistent with expectations: FOA provides high quality predictions when pass-through is perfectly observed, while (by design) informed simulation only partially mitigates functional form misspecification.[21]

Table II provides the median absolute prediction errors (MAPEs) generated by the different methodologies. As shown, the MAPEs of FOA tend to be an order of magnitude smaller than those of standard simulations,

[20] Two clarifications may assist in the interpretation of Figure 1. First, the post-merger prices are censored at 1.25 and, in some instances, the simulated price increases are well above this level. This may lead the figure to optically understate the degree of prediction error. Appendix Figure C.2 (See Supplementary Materials online) extends the range of the vertical axis to 1.50 to illustrate. Second, the figure is symmetric by construction. For example, the scatterplot for logit merger simulation when underlying demand is AIDS is the inverse of the scatterplot for AIDS merger simulation when underlying demand is logit.

[21] If, in our setting, one allowed the true underlying demand system to be identified from pass-through, then informed simulation would predict merger effects with zero prediction error. As discussed above, we consider this possibility to be unlikely outside our numerical experiments, as there is no reason that consumer decisions should be expected to conform to any of the standard (tractable) models.

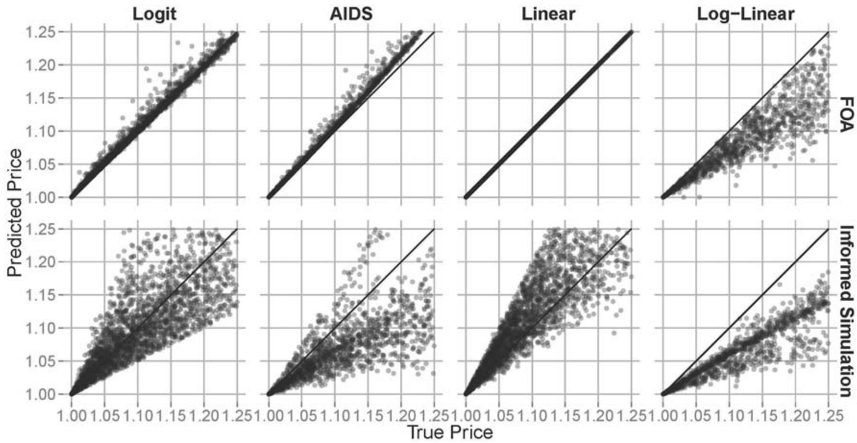

Figure 2
Prediction Error from FOA and Informed Simulation

Notes: The scatter plots characterize the accuracy of FOA and informed simulation when the underlying demand system is logit, almost ideal, linear, and log-linear. Each dot represents the first firm's predicted and actual post-merger prices for a given draw of data.

provided there is some functional misspecification in the simulation. The improvements in accuracy with informed simulation are more modest – the MAPEs are close to what arises under the least consequential functional form misspecification. The similarity exists because informed simulation largely serves to select among the misspecified demand systems in order to minimize prediction error.

Panel A of Table III provides the frequencies with which FOA has smaller absolute prediction error (APE) than standard merger simulations. As shown, FOA is more accurate than standard AIDS, linear and log-linear simulations for 99%, 89% and 100% of the mergers, respectively, when the underlying demand system is logit. Similarly high frequencies arise with the other demand systems. Aggregating across the four demand

Table II
Median Absolute Prediction Error

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
FOA	0.002	0.018	0.000	0.101
Informed Simulation	0.020	0.078	0.019	0.133
Logit Simulation	0.000	0.088	0.016	0.207
AIDS Simulation	0.090	0.000	0.103	0.122
Linear Simulation	0.016	0.102	0.000	0.220
Log-Linear Simulation	0.215	0.122	0.228	0.000

Notes: The table provides the median absolute prediction error of FOA, informed simulation and standard simulations. Pass-through is assumed to be observed perfectly.

TABLE III
FREQUENCY THAT PASS-THROUGH IMPROVES ACCURACY

Panel A: First Order Approximation

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
Logit Simulation	–	90.2%	100%	95.3%
AIDS Simulation	99.4%	–	100%	53.1%
Linear Simulation	89.2%	92.0%	–	93.9%
Log-Linear Simulation	100%	97.7%	100%	–

Panel B: Informed Simulation

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
Logit Simulation	–	88.1%	86.3%	97.8%
AIDS Simulation	94.4%	–	98.4%	81.3%
Linear Simulation	80.9%	70.4%	–	90.8%
Log-Linear Simulation	100%	92.4%	100%	–

Notes: Panel A shows the fraction of data draws for which FOA has a smaller absolute prediction error than standard merger simulations in predicting firm 1's price change. Panel B shows the same statistic for informed simulation, but allows for ties.

systems, FOA is more accurate than standard simulations for 93% of the mergers considered, provided there is some functional form misspecification. Panel B shows that informed simulation also yields more accurate predictions for the bulk of mergers when compared to standard simulation. Ties occur here, by design, because informed simulation is always identical to one of the misspecified simulations. We report the fraction of mergers for which informed simulation has an APE that is at least as small as standard merger simulations. Aggregating across the systems, informed simulation is at least as accurate as the standard merger simulations in 90% of the mergers, provided some misspecification exists, and more accurate in 57% of the mergers.

One measure of whether these improvements in accuracy are economically meaningful is whether they would improve enforcement decisions made on the basis of the predicted price effects. To explore this, we examine the propensity of the prediction methodologies to produce 'false positives' and 'false negatives.' We define false positives as price increase predictions that exceed ten per cent when the true effect is less than ten per cent. We define false negatives analogously.[22] The results are summarized in Table IV. FOA generates both few false positive and few false negatives,

[22] We select a ten per cent threshold solely based on the empirical distribution of true prices changes: in each demand system, many mergers produce true price effects both above and below this threshold. We have examined other thresholds and the qualitative results are unaffected.

TABLE IV
TYPE I AND II PREDICTION ERROR

Panel A: Frequency of False Positives (Type I)

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
FOA	1.6%	1.5%	0.0%	0.9%
Informed Simulation	6.7%	2.4%	11.3%	0.5%
Logit Simulation	–	0.3%	9.6%	0.0%
AIDS Simulation	23.2%	–	30.4%	0.0%
Linear Simulation	2.2%	0.0%	–	0.0%
Log-Linear Simulation	34.4%	12.6%	41.8%	–

Panel B: Frequency of False Negatives (Type II)

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
FOA	0.0%	1.0%	0.0%	14.5%
Informed Simulation	8.2%	19.0%	1.5%	16.9%
Logit Simulation	–	25.0%	2.2%	39.1%
AIDS Simulation	0.2%	–	0.0%	13.3%
Linear Simulation	9.6%	32.4%	–	46.8%
Log-Linear Simulation	0.0%	0.0%	0.0%	–

Notes: Panel A shows the fraction of data draws for which the true price change in firm 1's price is less than 10 per cent but the prediction exceeds 10 per cent. Panel B the fraction of data draws for which the true price change exceeds 10 per cent but the prediction is less than 10 per cent. FOA is calculated using the pass-through that arises in the pre-merger equilibrium.

while standard merger simulations yield either many false positives or many false negatives, provided there is some misspecification in functional form. Thus, for example, it is possible to generate conservative predictions of merger price effects with linear and logit simulations, but if such simulations receive weight in enforcement decision-making then a nontrivial number of anticompetitive mergers would proceed. Informed simulation also tends to improve the balance of false positives and negatives, albeit to a lesser extent than FOA.

We turn now to prediction error conditional on the magnitude of the true merger price effect. To implement, we regress APE on the price effect using nonparametric techniques, and examine the obtained fitted values.[23] Figure 3 plots the results when the true underlying demand system is almost ideal (the other demand systems produce qualitatively similar patterns). We draw two main sets of conclusions. First, while prediction error becomes larger as the true price effect grows regardless of prediction methodology, this relationship is much stronger for simulation than for FOA. It is intuitive that the consequences of functional form misspecification should increase as the counterfactual prices become further from the initial

[23] We use kernel-weighted local polynomial regressions with the standard Epanechnikov kernel.

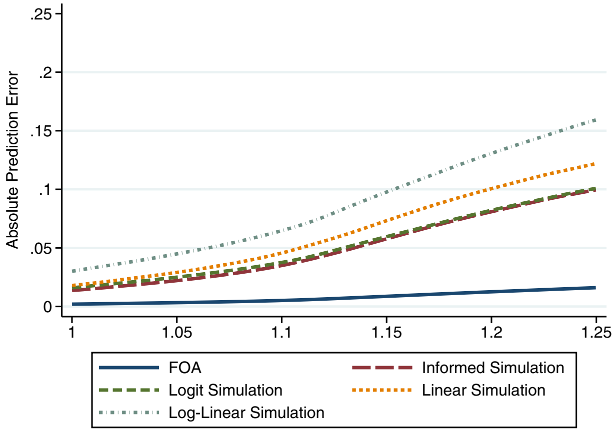

Figure 3
Absolute Prediction Error for Conditional on the Price Effect [Colour figure can be viewed at *wileyonlinelibrary.com*]
Notes: The panels plot fitted value for absolute prediction error (APE) obtained with non-parametric regressions of APE on the true price effect. The underlying demand system is almost ideal.

equilibrium. The results indicate, however, that with FOA the scale of this problem is much less severe. This suggests that knowledge of pass-through is especially valuable for understanding counterfactual scenarios that involve large price changes. Second, the relative accuracy of FOA is maintained even as the true merger price effects become small. We are unable to identify any range of prices for which standard merger simulation is as accurate as FOA, provided that pass-through is perfectly observed.

V(ii). *Measurement Error and Bias in Pass-Through*

Table V shows the MAPEs that arise with FOA and informed simulation when pass-through is observed with measurement error or bias. The accuracy of FOA deteriorates with the magnitude of measurement error regardless of the underlying demand system. This is consistent with expectation, as pass-through determines the extent to which the opportunity costs created by the merger affect prices, and therefore measurement error affects the accuracy with which merger pass-through can be recovered. However, even when the pass-through data are quite noisy, prediction error usually is smaller than under standard merger simulation, provided that some misspecification exists (see Table II). The small amount of bias introduced also increases MAPE in most cases. The effect of bias is more easily seen graphically, and we return to this shortly.

The accuracy of informed simulation also deteriorates with measurement error in pass-through, but less quickly relative to FOA. This is because

TABLE V
MAPE WITH IMPERFECT PASS-THROUGH DATA

Panel A: First Order Approximation

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
30% Measurement Error	0.013	0.032	0.009	0.102
60% Measurement Error	0.023	0.071	0.019	0.120
90% Measurement Error	0.038	0.118	0.034	0.159
15% Downward Bias	0.015	0.021	0.014	0.142
15% Upward Bias	0.019	0.067	0.015	0.069

Panel B: Informed Simulation

	Underlying Demand System:			
	Logit	AIDS	Linear	Log-Linear
30% Measurement Error	0.020	0.078	0.019	0.131
60% Measurement Error	0.021	0.081	0.019	0.132
90% Measurement Error	0.022	0.088	0.019	0.138
15% Downward Bias	0.016	0.084	0.018	0.134
15% Upward Bias	0.026	0.073	0.020	0.129

Notes: The table shows the median absolute prediction error that arises with (i) FOA supported by cost pass-through observed within 30%, 60%, and 90% of its true value; (ii) FOA supported by cost pass-through with 15% downward and upward biases; and (iii) informed simulation, as defined by the most accurate misspecified simulation model.

measurement error only rarely causes a change in the misspecified model selected for use in the informed simulation. For example, when underlying demand is logit, the linear demand system is selected for use in simulation for 74.4% of mergers if there is no measurement error, and for 73.7% of mergers if pass-through is observed within 90% of its true value. Due to the robustness of this selection routine, the accuracy of informed simulation exceeds that of FOA when measurement error in pass-through is large.[24] Bias at the level considered also does not affect the selection routine substantially, and so the MAPE of informed simulation is mostly unaffected.

Figure 4 provides scatter plots of the prediction error with FOA.[25] The presence of measurement error in pass-through leads to a greater spread of FOA predictions, and the spread increases in the magnitude of the measurement error. Predictions remain centered around zero, however, so measurement error does not lead to systematic over-prediction or under-prediction. The predicted price effects of FOA are muted when cost pass-through is observed with downward bias, and amplified when pass-through

[24] Note that our results are in part dependent on the menu of demand systems we have chosen for our exercise.

[25] The figure shows the cases in which cost pass-through is observed within 30% and 90% of its true value. We omit the case of 60% due to space considerations.

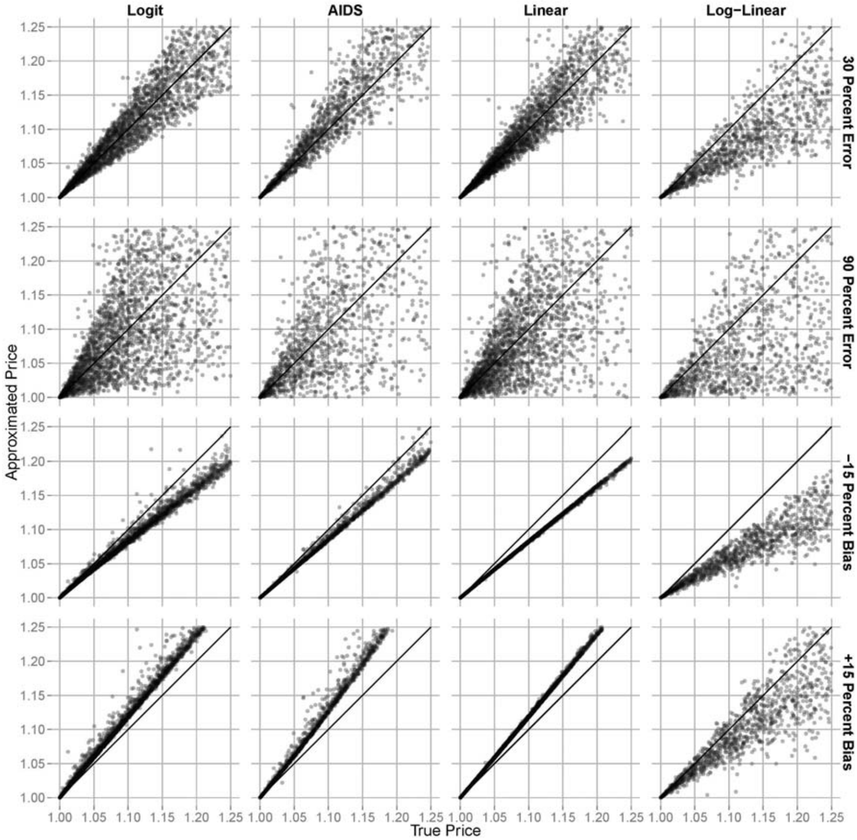

Figure 4
Prediction Error from FOA with Imperfect Pass-Through Data

Notes: The scatter plots characterize the accuracy of FOA when the underlying demand system is logit, almost ideal, linear, and log-linear. Each dot represents the first firm's predicted and actual post-merger prices for a given draw of data. FOA is calculated based on pass-through observed within 30% and 90% of its true value (rows 1 and 2), and observed with 15% downward and upward biases (rows 3 and 4).

is observed with upward bias. Again this is consistent with the underlying economic theory. Lastly, we note that upward bias in pass-through reduces MAPE for the specific case of log-linear demand precisely because FOA otherwise understates price effects (e.g., see Figure 2).

Figure 5 provides the same scatter plots for informed simulation. Predictions are not centered around the true price effects. Nonetheless, the extent is visibly reduced relative to standard merger simulations (see Figure 1), and the magnitude of measurement error does not lead to a greater spread of predictions. The presence of bias at the level examined does not affect

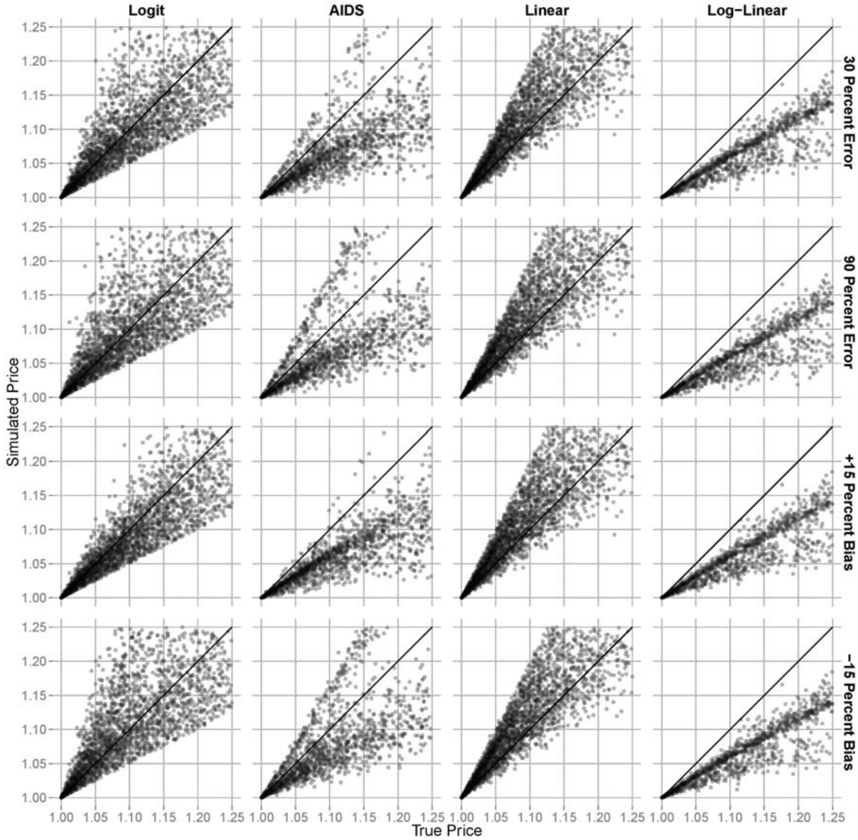

Figure 5

Prediction Error from Informed Simulation with Imperfect Pass-Through Data

Notes: The scatter plots characterize the accuracy of informed simulation when the underlying demand system is logit, almost ideal, linear, and log-linear. Each dot represents the first firm's predicted and actual post-merger prices for a given draw of data. Informed simulation is calculated based on pass-through observed within 30% and 90% of its true value (rows 1 and 2), and observed with 15% downward and upward biases (rows 3 and 4).

much the predictions with informed simulation, again because the selection routine that chooses the demand model proves to be robust. Robustness to measurement error derives from the way pass-through affects the predictions of informed simulation: pass-through has no direct effect because it is used only to select among demand schedules. This limits the influence of poorly measured pass-through terms that are difficult to reconcile with economic theory. The finding suggests that it is appropriate to interpret pass-through through an economic model if it is observed with significant measurement error.

VI CONCLUSION

The Monte Carlo experiments that we examine demonstrate that using pass-through to supplement information on demand elasticities can substantially improve the predictions of merger price effects. When pass-through is precise, predictions based on the first order approximation (FOA) of Jaffe and Weyl [2013] are tightly distributed around the true price effects, and avoid the prediction errors that arise in standard merger simulation models due to functional form misspecification. The predictive accuracy of FOA deteriorates with the degree of measurement error in pass-through. An alternative to FOA that entails using pass-through to select among functional forms for use in simulation also increases accuracy relative to standard merger simulation, and proves more robust to measurement error.

Because the results broadly suggest a potentially important role for pass-through in the evaluation of mergers and other counterfactual analyses, we conclude with a brief discussion about some of the difficulties that can arise in the estimation and interpretation of pass-through. First, MacKay *et al.* [2014] develop that econometric biases can plague reduced-form linear regressions of prices on cost shifters if pass-through is non-constant, even if standard orthogonality conditions hold. In such settings, the regression recovers the average effect of costs on prices, but this need not map into pass-through at any particular price point. The extent to which average pass-through is useful for counterfactuals is not established in our experiments here, and could be the focus of additional research. Second, menu costs, rule-of-thumb pricing, and nonlinear demand can also frustrate attempts to estimate pass-through, depending on the variation used to identify regression parameters, and they may also affect the derived theoretical relationship between local demand curvature and cost pass-through. Such forces may create a relevant distinction between long run and short run pass-through rates. This distinction is emphasized in the literature on asymmetric pass-through (e.g., Borenstein *et al.* [1997]; Peltzman [2000]) and increasingly is modeled explicitly (e.g., Nakamur and Zerom [2010]; Goldberg and Hellerstein [2013]), but more research on this subject would be valuable.

APPENDIX A: CALIBRATION DETAILS

We provide mathematical details on the calibration process in this appendix. To distinguish the notation from that of Section II, we move to lower cases and let, for example, s_i and p_i be the market share and price of firm i's product, respectively.[26] Recall that in the data generating process we randomly assign market shares among the four single-product firms and the outside good, draw the price-cost margin of the first firm's product from a uniform distribution with support over (0.2, 0.8), and normalize all prices to unity. The calibration process then obtains parameters for the

[26] We define market share $s_i = q_i / \sum_{j=1}^{N} q_j$, where q_i represents unit sales.

logit, almost ideal, linear, and log-linear demand systems so as to reproduce these draws of data.

Calibration starts with multinomial logit demand, the basic workhorse model of the discrete choice literature. The system is defined by the share equation

$$s_i = \frac{e^{(\delta_i - \alpha p_i)}}{1 + \sum_{j=1}^{N} e^{(\delta_j - \alpha p_j)}} \tag{A.1}$$

The parameters to be calibrated include the price coefficient α and the product-specific quality terms δ_i. We recover the price coefficient by combining the data with the first order conditions of the first firm. Under the assumption of Nash-Bertrand competition this yields:

$$\alpha = \frac{1}{m_1 p_1 (1 - s_1)} \tag{A.2}$$

where m_1 is the price-cost margin of firm 1. We then identify the quality terms that reproduce the market shares:

$$\delta_i = \log(s_i) - \log(s_0) + \alpha p_i \tag{A.3}$$

for $i = 1 \ldots N$. We follow convention with the normalization $\delta_0 = 0$. Occasionally, a set of randomly-drawn data cannot be rationalized with logit demand, and we replace it with a set that can be rationalized. This tends to occur when the first firm has both an unusually small market share and an unusually high price-cost margin.

The logit demand system often is criticized for its inflexible demand elasticities. Here, the restrictions on substitution are advantageous and allow us to obtain a full matrix of elasticities with a tractable amount of randomly drawn data. The derivatives of demand with respect to prices, as is well known, take the form

$$\frac{\partial q_i}{\partial p_j} = \begin{cases} \alpha s_i (1 - s_i) & \text{if } i = j \\ -\alpha s_i s_j & \text{if } i \neq j \end{cases} \tag{A.4}$$

We use the logit derivatives to calibrate the more flexible almost ideal, linear, and log-linear demand systems. This ensures that each demand system has the same first order properties in the pre-merger equilibrium, for a given draw of data.

The AIDS is written in terms of expenditure shares instead of quantity shares (Deaton and Muellbauer [1980]). The expenditure share of product i takes the form

$$w_i = \alpha_i + \sum_{j=0}^{N} \gamma_{ij} \log p_j + \beta_i \log(x/P) \tag{A.5}$$

where x is total expenditure and P is a price index. We incorporate the outside good as product $i = 0$ and normalize its price to one; this reduces to N^2 the number of price coefficients in the system that must be identified (i.e., γ_{ij} for $i, j \neq 0$). We further

set $\beta_i=0$ for all i, a restriction that imposes an income elasticity of unity. Under this restriction, total expenditures are given by

$$\text{(A.6)} \qquad \log(x)=(\tilde{\alpha}+u\tilde{\beta})+\sum_{k=1}^{N}\alpha_k\log(p_k)+\frac{1}{2}\sum_{k=1}^{N}\sum_{j=1}^{N}\gamma_{kj}\log(p_k)\log(p_j)$$

for some utility u. We identify the sum $\tilde{\alpha}+u\tilde{\beta}$ rather than $\tilde{\alpha}$, u, and $\tilde{\beta}$ individually.[27]

Given this structure, product i's unit sales are given by $q_i=xw_i/p_i$ and the first derivatives of demand take the form

$$\text{(A.7)} \qquad \frac{\partial q_i}{\partial p_j}=\begin{cases}\dfrac{x}{p_i^2}(\gamma_{ii}-w_i+w_i^2) & \text{if } i=j\\[2ex] \dfrac{x}{p_ip_j}(\gamma_{ij}+w_iw_j) & \text{if } i\neq j\end{cases}$$

The calibration process for the AIDS then takes the following four steps:

1. Calculate x and w_i from the randomly drawn data on market shares, using a market size of one to translate market shares into quantities.
2. Recover the price coefficients γ_{ij} for $i,j\neq 0$ that equate the AIDS derivatives given in equation (A.7) and the logit derivatives given in equation (A.4). Symmetry is satisfied because consumer substitution is proportional to share in the logit model. The outside good price coefficients, γ_{i0} and γ_{0i} for all i, are not identified and do not affect outcomes under the normalization the $p_0=1$. Nonetheless, they can be conceptualized as taking values such that the adding up restrictions $\sum_{i=0}^{N}\gamma_{ij}=0$ hold for all j.
3. Recover the expenditure share intercepts α_i from equation (A.5), leveraging the normalization that $\beta_i=0$. The outside good intercept α_0 is not identified and does not affect outcomes, but can be conceptualized as taking a value such that the adding up restriction $\sum_{i=0}^{N}\alpha_i=1$ holds.
4. Recover the composite term $(\tilde{\alpha}+u\tilde{\beta})$ from equation (A.6).

This process creates an AIDS that, for any given set of data, has quantities and elasticities that are identical in the pre-merger equilibrium to those that arise under logit demand. The system possesses all the desirable properties defined in Deaton and Muellbauer [1980]. Our approach to calibration differs from Epstein and Rubinfeld [2001], which does not model the price index as a function of the parameters, and from Crooke *et al.* [1999], which assumes total expenditures are fixed.

We turn now to the linear and log-linear demand systems. The first of these takes the form

[27] The price index P is defined implicitly by equation (A.6) as the combination of prices that obtains utility u given expenditure x. A formulation is provided in Deaton and Muellbauer [1980].

$$q_i = \alpha_i + \sum_j \beta_{ij} p_j, \tag{A.8}$$

The parameters to be calibrated include the firm specific intercepts α_i and the price coefficients β_{ij}. We recover the price coefficients directly from the logit derivatives in equation (A.4). We then recover the intercepts to equate the implied quantities in equation (A.8) with the randomly drawn market shares, again using a market size of one. Of similar form is the log-linear demand system:

$$\log(q_i) = \gamma_i + \sum_j \epsilon_{ij} \log p_j \tag{A.9}$$

where the parameters to be calibrated are the intercepts γ_i and the price coefficients ϵ_{ij}. Again we recover the price coefficients from the logit derivatives (converting first the derivatives into elasticities). We then recover the intercepts to equate the implied quantities with the market share data. This process creates linear and log-linear demand systems that, for any given set of data, have quantities and elasticities that are identical to those of the calibrated logit and almost ideal demand systems in the pre-merger equilibrium.

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SUPPORTING INFORMATION

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Appendix B. Merger Pass-through
Appendix C. Additional Figures